Symmetry TFTs from M/string theory

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based on

[2112.02092 with Apruzzi, Bonetti, García Etxebarria and Schafer-Nameki ’21]
[Work to appear with García Etxebarria ‘22]

Generalised symmetries

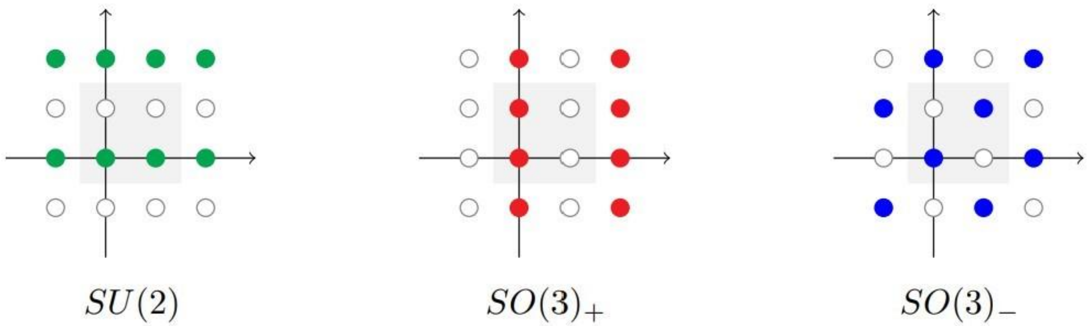
To specify a QFT → specify its local and **global structure**

[Gaiotto, Kapustin, Seiberg, Willett '14]

Dirac charge quantisation → choices of global structures

[Aharony, Seiberg, Tachikawa, '13]

Example: 4d SYM with Lie algebra *su(2)*

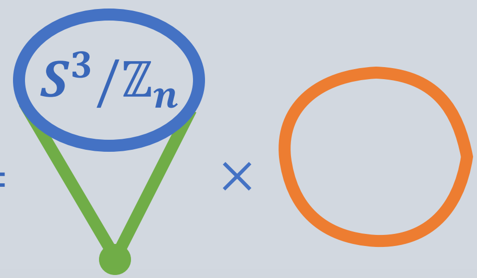


Generalised symmetries from string theory

[García Etxebarria, Heidenreich, Regalado ’19]

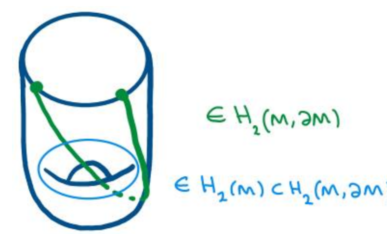
How about theories with no gauge theory description?

M-theory on M_{11} —— Geometric engineering ⟶ Field theory M_7

$M_{11} = \mathbb{C}^2/\mathbb{Z}_n \times M_7 =$ 

Branes wrapping non-compact cycles ⟶ Defects

Branes wrapping compact cycles ⟶ Dynamical operators



Defect Group: $D = H_2(\mathbb{C}^2/\mathbb{Z}_n, S^3/\mathbb{Z}_n)\,/H_2(\mathbb{C}^2/\mathbb{Z}_n) = \mathbb{Z}_n \oplus \mathbb{Z}_n$

Flux non-commutativity

[Witten '98], [Diaconescu, Freed, Moore '03], [Moore '04], [Freed, Moore, Segal, '06]

M-theory fluxes ⟶ Symmetry generators acting on defects

M-theory fluxes $\boldsymbol{\phi}$ do not commute: $\boldsymbol{\phi_\sigma \, \phi_\tau = e^{2\pi i \, L(\sigma,\tau)} \phi_\tau \, \phi_\sigma}$

⟶ Cannot realise all symmetry generators

⟶ Pick a set of commuting fluxes

Choices of commuting fluxes ⟷ Choices of global structures

Example: 4d SYM with Lie algebra *su(2)*

$\mathrm{L} \subset D = \mathbb{Z}_2^{(1)} \oplus \mathbb{Z}_2^{(2)}$: $\mathrm{L} = \mathbb{Z}_2^{(1)}, \mathbb{Z}_2^{(2)}, \mathbb{Z}_2^{Diag}$ give the 3 possible global structures

BF theory

$$S_{BF}(B,A) = l \int_{M^{D+1}} B_{q+1} \wedge \underbrace{F}_{= dA_{D-q-1}}$$

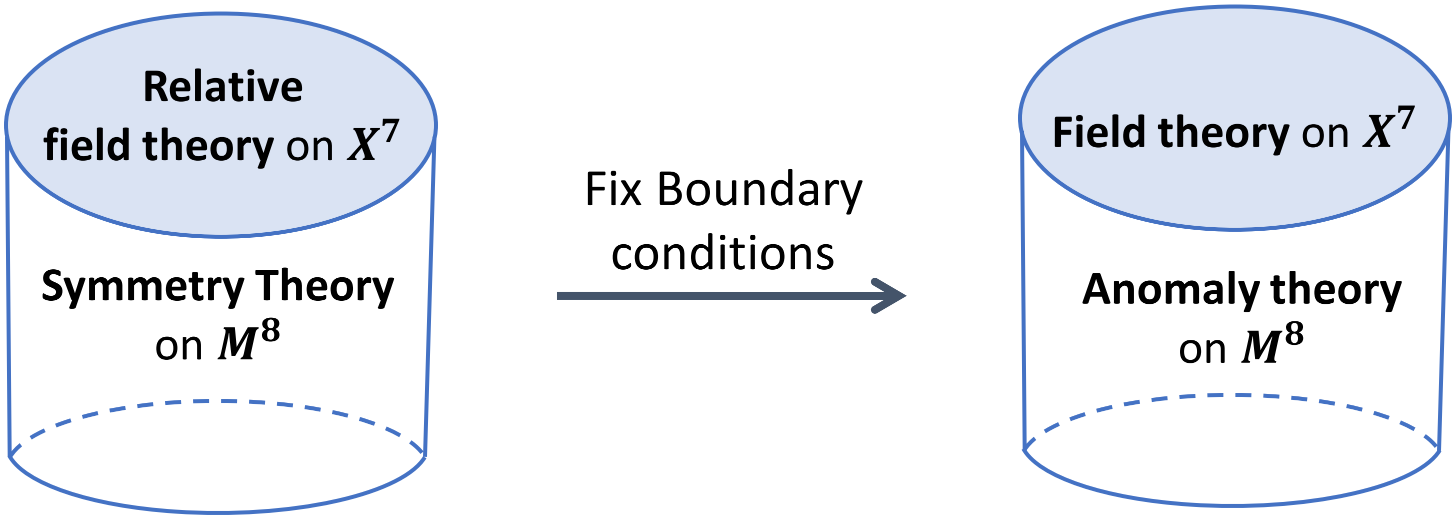
B: background for the q-form electric symmetry

A: background for the (D-q)-form magnetic symmetry

For $l > 1$, $\boldsymbol{S_{BF}(B,A)}$ is a non-invertible theory, whose state space reproduces the **choices of global structure** for field theory on ∂M.

Symmetry Theory

[Freed], [Apruzzi, Bonetti, García Etxebarria, Hosseini and Schäfer-Nameki '21]



Symmetry Theory encodes

- The **BF theory**
- All the possible **anomaly theories**
- **Higher structures**

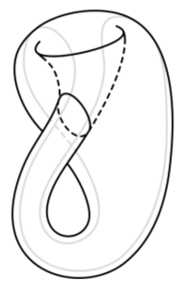
for a relative field theory

Differential cohomology

[Cheeger and Simons '85], [Hopkins and Singer '02]

Usually we take the fields $A \in \Omega^2(X)$ to be differential forms, but we can have **topologically non-trivial fields** encoding **torsion** in X.

Klein Bottle has a torsional 1-cycle $H_1(X;\mathbb{Z}) = \mathbb{Z}_2$



Thus, promote $\breve{A}$ to be a class in **differential cohomology**

$$\breve{A} = \big(F(A), h(A), I(A)\big) \in \Omega_{\mathbb{Z}}{}^2(X) \times H^1(X;\mathbb{R}) \times H^2(X;\mathbb{Z}) = \breve{H}^2(X)$$

where $\delta h(A) = F(A) - I(A)$.

Anomalies

[Cvetič, Dierigl, Lin and Zhang '21], [Apruzzi, Bonetti, García Etxebarria, Hosseini and Schäfer-Nameki '21]

Chern-Simons for the M-theory topological action ($\int G_4 G_4 C_3$)

$$S_{top} = -\frac{1}{6}\int_{M^{11}=X^8\times S^3/\mathbb{Z}_n} \breve{G}_4 \cdot \breve{G}_4 \cdot \breve{G}_4 = \frac{1}{2}\int_{X^8} \breve{\gamma}_4 \cdot \breve{\gamma}_4 \cdot \breve{\xi}_1 - CS[S^3/\mathbb{Z}_{\boldsymbol{n}}, \breve{t}_2]\int_{X^8} \breve{\gamma}_4 . \breve{B}_2 . \breve{B}_2$$

$\breve{B}_2 \in \breve{H}^2(X^8)$, $\quad \breve{\gamma}_4 \in \breve{H}^4(X^8)$, $\quad \breve{\xi}_1 \in \breve{H}^1(X^8)$

Here, we could only derive the BF theory by looking at the non-commutativity of the operators.

Puzzle: How do we obtain the BF action directly from dimensional reduction (of what)?

Chiral scalar

Chern-Simons for the **kinetic action** for a **self-dual field**. [Belov, Moore '06]

Consider a self-dual chiral scalar $*\phi = \phi$ in 2d. Naively, the action is $\int_{M^2} d\phi \wedge d * \phi$, but doesn't encode self-duality.

Couple ϕ to a background field A

$$S = \int_{M^2} (d\phi - A) \wedge *(d\phi - A) - \int_{M^2} d\phi \wedge A$$

Under gauge trans. $\delta A = d\lambda$ and $\delta\phi = \lambda$

$$\delta S = \int_{M^2} \lambda \wedge dA$$

This is the change under $\delta A = d\lambda$ of the CS action

$$CS(A) = \int_{N^3} A \wedge dA \xrightarrow{\text{For } \breve{A} \text{ topologically non-trivial}} CS(\breve{A}) = \int_{N^3} \breve{A} \cdot \breve{A}$$

BF theory

Chern-Simons for the **kinetic action** for a **self-dual field**.

In M-theory $\breve{G}_4$ and $\breve{G}_7$ fluxes form a pair $\breve{G} = (\breve{G}_4, \breve{G}_7)$ which is a self-dual field.

[Freed '00]

Define Chern-Simons on N^{12} for $\breve{G}$ by coupling it to a background field with current $\breve{B} = (\breve{B}_5, \breve{B}_8)$ which extends to N^{12}.

[Belov, Moore '06]

$$\frac{1}{2}\int_{N^{12}} \breve{B} \cdot \breve{B} = \int_{N^{12}} \breve{B}_8 \cdot \breve{B}_5$$

BF theory

Consider $N^{12} = S^3/\mathbb{Z}_n \times W^9$ with $\partial W^9 = X^8$

$$S_{BF}(\breve{B}_2, \breve{C}_2) = \int_{N^{12}} \breve{B}_8 \cdot \breve{B}_5 = \int_{S^3/\mathbb{Z}_n} \breve{t}_2 \cdot \breve{t}_2 \int_{W^9} d\breve{B}_2 . d\breve{C}_5 = L(\breve{t}_2, \breve{t}_2) \int_{X^8} \breve{B}_2 . d\breve{C}_5$$

using Stokes theorem.

$\breve{B}_2 \in \breve{H}^2(X^8)$: background for the electric 1-form symmetry,

$\breve{C}_5 \in \breve{H}^5(X^8)$: background for the magnetic 4-form symmetry.

Have a choice of global structure as cannot realise both symmetries.

Conclusion

- **Global structures** and **anomalies** in symmetry theory from **11d supergravity**

- Formulation of supergravity in **differential cohomology** to include **torsion**

- Goal is to **derive the full symmetry theory**, including **categorical** cases (higher groups and non-invertibles). Many works on the topic including [Del Zotto, Heckman, Meynet, Moscrop, and Zhang '22], [Del Zotto, García Etxebarria, Schäfer-Nameki '22], [Cvetič, Heckman, Hübner, Torres '22], [Del Zotto, García Etxebarria '22], [Argyres, Martone and Ray '22], [Bhardwaj, Bottini, Schäfer-Nameki, Tiwari '22], [Bhardwaj, Bullimore, Ferrari and Schäfer-Nameki '22], … but much remains to be done!

Applications:

- No global symmetries and weak gravity conjectures [Heidenreich, McNamara, Montero, Reece, Rudelius, and Valenzuela '21], [Arias-Tamargo and Rodriguez-Gomez '22]
- Constraints on RG flows [Del Zotto, García Etxebarria, and Hosseini '20]
- Dualities [Del Zotto and Ohmori '20]
- Confinement [Apruzzi, van Beest, Gould, and Schäfer-Nameki '21]
- Non-invertible symmetries in standard model [Choi, Lam, and Shao '22], [Cordova, Clay and Ohmori, Kantaro '22]